RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2018-21]
B.A./B.Sc. FIRST SEMESTER (July – December) 2018
Mid-Semester Examination, September 2018

Date : 24/09/2018 **PHYSICS (Honours)**
Time : 11 am – 1 pm Paper : I Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(*Answer any four questions*) [4×5]

1. a) Show that $\vec{A}\bullet\dfrac{d\vec{A}}{dt}=A\dfrac{dA}{dt}$ [2]

   b) Show that the acceleration $\vec{a}$ of a particle which travels along a space curve with velocity $\vec{v}$ is given by $\vec{a}=\dfrac{dv}{dt}\hat{T}+\dfrac{v^2}{\rho}\hat{N}$; where $\hat{T}$ is the unit tangent vector to the space curve and $\hat{N}$ is its unit principal normal, and ρ is the radius of curvature. [3]

2. a) What is the physical significance of gradient of some scaler function ϕ (x,y,z) ? [2]

   b) Find the equation of the tangent plane and normal line to the surface $z = x^2+y^2$ at the point (2,-1,5) [3]

3. State and Prove the divergence theorem. [5]

4. Let $\vec{A}=2yz\hat{i}-(x+3y-2)\hat{j}+(x^2+z)\hat{k}$. Evaluate $\iint\limits_S(\vec{\nabla}\times\vec{A})\bullet\overrightarrow{ds}$ over the surface of intersection of the cylinders $x^2+y^2=a^2$, $x^2+z^2= a^2$ which is included in the first octant. [5]

5. Solve $\dfrac{dy}{dx}+\dfrac{y}{x}=2x^3y^4$. [5]

6. If Y_1= x and Y_2= x e^x are two solutions of the homogeneous differential equation associated with $x^2y^{//}$ - $(x^2+2x)y^{/}$ + (x+2) y = x^3

   a) Show that Y_1 and Y_2 are linearly independent. [2]

   b) Find the general solution of the differential equation. [3]

7. Using the method of separation of variables solve. [5]

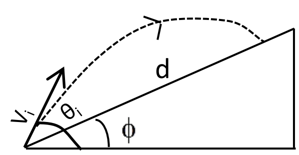
$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=u \text{ where } u(o,y)=3e^{-y}-e^{-5y}$$

Group – B

(*Answer any three questions*) [3×5]

8. Find the expression of velocity and acceleration of a particle in cylindrical coordinates (ρ,ϕ,z). [5]

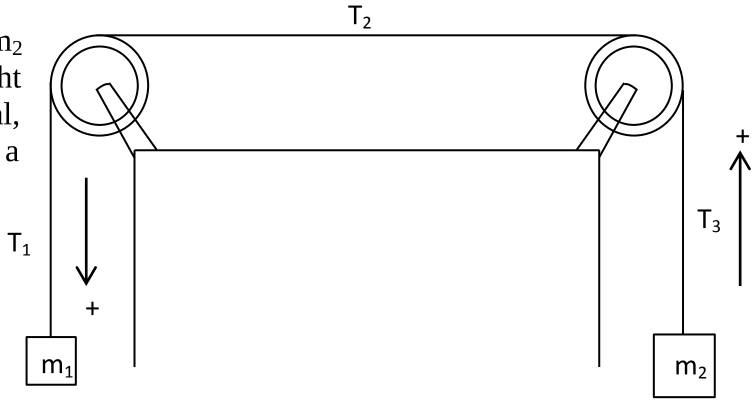
9. A projectile is fixed up an incline (incline angle θ) with an initial speed v_i at an angle θ_i with respect to the horizontal $(\theta_i>\phi)$, as shown in figure. Show that the projectile travels a distance d up the incline. where $d=\dfrac{2v_i^{\,2}\cos\theta_i\sin(\theta_i-\phi)}{g\cos^2\phi}$ [5]



10. a) A rocket is moving in free space by ejecting gases at constant speed with v_e respect to the rocket. Set the equation of motion for the rocket and solve it to find an expression of motion for the rocket and solve it to find an expression of velocity of the rocket in terms of its initial and final mass. [4]

b) Explain briefly how a multistage rocket can be useful in increasing its final velocity. [1]

11. Two blocks having masses m_1 and m_2 are connected to each other by a light cord that passes over two identical, frictionless pulleys, each having a moment of inertia I and radius R, as shown in figure. Find



a) the acceleration of each block.

b) the angular acceleration of each pulley .

c) The tensions T_1, T_2 and T_3 in the cord. (Assume no slipping between cord and pulleys.) [5]

12. a) Show for an inverse square attractive force the orbit of a particle will be a conic section. [2]

b) A satellite has largest and smallest orbital speeds given by U_{max} and U_{min} respectively. Prove that the ecentricity of the orbit in which the satellite moves is equal to $\varepsilon = \frac{U_{max} - U_{min}}{U_{max} + U_{min}}$ [3]

Group – C

(*Answer any three questions*) [3×5]

13. a) "All simple harmonic motions are periodic , but all periodic motions are not simple harmonic"—commment. [1]

b) One end of a uniform spring of spring constant K and a finite mass m is attached to a rigid wall. The other end of the spring is attached to a body of M (>m) placed on a horizontal friction less surface. Show that the angular frequency of horizontal harmonic oscillation is $\left[K \big/ \left(M + m/3\right)\right]^{1/2}$ [4]

14. Establish the equation of motion of a damped harmonic oscillator subjected to a resistive force that is proportional to the first power of its velocity. If the damping is less than critical, show that the motion of the system is oscillatory with its amplitude decreasing exponentially with time. [5]

15. Obtain the expression for velocity amplitude of the forced oscillator. What is the velocity of 'ω' for velocity resonance? Show that at velocity resonance, the amplitude V_0 of the velocity equals F_o/p. Also, show that the velocity of the oscillator is in phase with the driving force at velocity resonance. [All symbols carry their usual meaning]. [5]

16. An object of mass 2 kg hangs from a spring of negligible mass. The spring is extended by 2.5 cm where the object is attached. The top end of the spring is oscillated up and down in SHM with an amplitude of 2 mm. The qualify factor (Q) of the system is 20. Take g = 10 ms^{-2}.

a) What is the angular frequency ω_0 of the free undamped oscillation.

b) What is the amplitude of forced oscillation at $\omega = \omega_0$?

c) What is the mean power input to maintain the forced oscillation at angular frequency (ω) are present greater than ω_0?

d) What is the mean power loss by friction at this frequency ? [5]

17. Obtain frequencies and configuration of the normal modes of the oscillation of two pendulums of equal string lengths and unequal masses with their bobs connected by means of an elastic mass less spring. Find also the general solution of the individual displacement in terms of normal mode solution. [5]

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